Benders Decomposition for Capacity Expansion Planning with Network Constraints and Uncertain Demand: the Spanish Case

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***Abstract*— This paper presents a stochastic model for generation expansion planning in a power system with network constraints and uncertain demand. A DC representation of the network is assumed. To make the problem computationally feasible when applied to the Spanish power system, a bi-level Benders decomposition has been developed. The main results of this application prove the robustness and the efficiency of the planning in comparison with other deterministic solutions. Outputs are the capacity to install of each technology (including renewable assets) in each year and in each Spanish Autonomous Community. The convergence of the Benders algorithm has also been tested.**

***Keywords*—Generation expansion planning, Benders decomposition, stochastic optimization, network constraints, RES integration.**

***JEL codes* – O21, Q55, L94, C61, D81.**

NOMENCLATURE

<u>*Indices*</u>

t	Technologies
a	Years
n, m	Nodes
s	Stochastic scenarios
l,j	Benders iterations

<u>*Model parameters*</u>

ϕ	Discount rate (%)
$CI_{t,a}$	Investment cost (€/MW)
$CP_{t,a}$	Production cost (€/MWh)
CNQ	Cost of non-supply demand (€/MWh)
$D_{n,a}$, $D_{n,s,a}$	Inelastic demand at node n (MWh)
$S_{n,a}$, $S_{n,s,a}$	International exchanges at node n. Positive values are imports (MWh)
$C^{U}_{t,a}$	Capacity of each unit of technology t (MW)
$Q_{t,n,a}$	Maximum capacity to install of technology t by year a (MW)
$Q_{n,m,a}$	Capacity of the transmission lines (MW)
$Q^{I}_{t,n}$	Initial install capacity of technology t (MW)
$Di_{t,n,a}$	Outage rate of each technology t. It also models the utilization factor for renewable technologies (p.u.)
$Ci_{t,n,a}$	Annual closure rate of plants of each technology t that were planned before the first planning year (p.u.)
$X_{n,m,a}$	Reactance of line n to m (p.u.)
$P_{n,s,a}$	Probability of each demand scenario (%)

<u>*BD algorithm parameters*</u>

δ^{j}	BD cut's type for each iteration (0=feasibility cuts,1=optimality cuts)
RF^{j}	Sub-problem objective value (€)
$\pi^{j}_{t,n,s,a}$	Dual variable of the maximum production constraint (€/MWh)
$QA^{j}_{t,n,a}$	Cumulative installed capacity (MW)
$NG^{j}_{t,n,a}$	Number of units to install (1,..,∞)
L, U	Lower and upper BD bounds (€)
ε	Benders tolerance (%)
IT	Benders maximum iterations (1,..,∞)

<u>*Continuous variables*</u>

c, mc, sc	Problem, master problem and sub-problem total costs (€)
$rf(\cdot)$	Benders resource function (€)
$f_{n,m,a}$, $f_{n,m,s,a}$	Power flow from node n to m. Positive values are imports (MW)
$\theta_{n,a}$, $\theta_{n,s,a}$	Phase angle at node n (rad)
$qp_{t,n,a}$, $qp_{t,n,s,a}$	Production of technology t (MWh)
$qi_{t,n,a}$, $qi_{t,n,s,a}$	New installed capacity (MW)
$qa_{t,n,a}$, $qa_{t,n,s,a}$	Cumulative installed capacity (MW)
$nq_{n,a}$, $nq_{n,s,a}$	Non-supply demand at node n (MWh)
$cm_{n,a}$, $cm_{n,s,a}$	Marginal cost at node n (MWh)

Integer variables

$ng_{t,n,a}$	Number of units to install of technology t (1,..,∞)

I. Introduction

The aim of long-term generation expansion problems, in the framework of power systems, consists in selecting the most adequate technologies to invest in, the amount of new generation capacity to install, and the sites for building the new generation plants. To do so, physical, geographical and economic criteria must be applied while ensuring that the total installed capacity adequately meets the expected demand in a long-term horizon.

Several approaches and techniques for the generation expansion problem are proposed in the literature, but can be classified under two main groups: models for liberalized systems (see [1] for a review), and models that assume centralized systems, sometimes just to avoid the complexity of oligopolistic modeling (see [2] and [3]). Within the first group (see [4] for a deeper classification of this group), several models consider the optimization of only one company, leading to MPEC structures ([5]). For example, [6] obtains an MPEC that represents the static bi-level capacity expansion problem of a company when the demand is uncertain. Investments and productions are decided in the upper level, while the lower level clears the market. The problem of one company, comparing Cournot and Stackelberg approaches ([7]), is analyzed in [8] taking into account hydro and pumped-hydro constraints. Reference [9] describes a bi-level model for the investment decisions of one company assuming a perfectly competitive market, and considering uncertainty not only in the demand but also in the competitors' generation capacities. A generation planning model is proposed in [10] when several agents are competing. To find a solution, [10] assumes a centralized entity in charge of the evaluation of each company planning in an iterative process. Each evaluation provides information regarding the competitors' behavior, which is used to maximize all the profit functions in the sense of Nash. Reference [11] presents a generation planning involving decisions on new units' construction by applying a Cournot model ([12]) at a single point in time.

EPEC structures ([13]) appears also in expansion models under competition. Reference [14] proposes an EPEC for the generation expansion considering one future year, which is linearized and solved using Mixed Integer Linear Programming ([15]). In [16] two bi-levels expansion problems are proposed under perfect and Cournot competitions, discussing existence and uniqueness issues. An EPEC that takes into account hydro power, demand and competitors investments uncertainty is described in [1]. Other realistic details such as the introduction of capacity mechanisms for financial hedging are also considered in [1].

The second group (centralized models) is very well studied in the literature (see [2] for metaheuristic techniques to solve the involved models, encompassing Genetic Algorithms, Expert Systems, Fuzzy Programming, Artificial Neural Networks, Analytic Hierarchy Processes, Network Flows, and Simulated Annealing). Unlike the game-based models of the first group (for liberalized systems), these centralized models can easily consider transmission network constraints (equilibrium existence is typically lost in game-based models including these constraints, see [17]). In [18] a robust generation expansion planning is obtained considering a set of probable demand scenarios. A MPEC model is presented in [19] to obtain the generation investments in the upper level by minimizing total costs, subject to a lower level for the market clearing under different load and wind conditions. References [3], [20] and [21] describe several multiobjective models for the expansion planning, obtaining Pareto solutions when cost, environmental impact and several types of risks are optimized simultaneously by a centralized entity.

In general, the main limitations in capacity expansion models are the size and complexity of the problems to be solved. These two aspects depend on both the horizon and the degree of detail with which the system is represented[1]. To solve this, different resolution methods consisting in splitting the problem in subproblems solved by stages, have been proposed. The method most used is Benders Decomposition (BD; see [22] and the annex for a review), that can be applied when the problem to be solved has a special block structure. Under this structure, BD is able to divide a very dense problem into different linked subproblems, and obtain the optimal solution iteratively. The model structure required to apply BD is often presented in applications that lead to stochastic programming, as it is the case of this paper.

Different applications of BD to electric power systems can be found in the literature. In [23] a model that combines the application of Genetic Algorithms and BD is proposed for the capacity expansion problem. Although this model takes into account the availability of the generation units, it does not consider uncertainty in the inputs. In [24] BD is applied to solve transmission expansion planning. Generation expansion is not optimized in this case. BD is also applied in [25] to optimize the capacity expansion when considering some probabilistic reliability constraints. The operation for a fixed capacity is optimized in the subproblem, while optimal

[1]Models for liberalized system also have an additional drawback related with the complexity of the equilibrium conditions to be solved.

capacity investments with yearly transversal constraints result from the master problem.

In this paper, a model for planning capacity expansion considering stochastic demand is proposed. A robust expansion plan against this uncertainty is obtained, similarly to [18] but considering the effect of network constraints. Only the expansion in generation technologies is modeled. Therefore, transmission lines capacities are considered inputs (see [24] for a possible extension to transmission expansion). As in the centralized models reviewed (see for example [8]), the model minimizes the expected total system cost, taking into account investment and expected operation costs. The main results are the total capacity to install, which is given by technology and node, the flows along each transmission line, the production of each technology and the electricity marginal cost at each node.

The main contribution of this paper is the application of the proposed model to the Spanish electric system, considering each Spanish Autonomous Community (CA)[2] as a node of the network, estimating the thermal limits of each line based on historical data from different public web pages, and their reactance based on [26]. Though Spanish market does not has nodal prices, our model can serve as a valid representation of the daily-ahead Spanish market when combined with the impact of the technical constraints market ([27]). For computational reasons, a bi-level BD is used to solve the proposed application, where investments decisions are optimized in the master problem while generation units operation is solved in the sub-problem.

This paper is organized as follows. Section II presents the formulation of the proposed generation expansion model without considering demand uncertainty, and when this uncertainty is included in the model. Section III exposes the application of the bi-level BD technique to the model. Section IV analyses a realistic case study. Finally, some conclusions are presented in the last section. The annex describes the mathematical formulation and an overview of the bi-level BD technique applied.

II. The Generation Expansion Problem with Network Constraints

A. The deterministic case

If the demand is deterministic and unitary durations for each year are assumed (hereinafter by simplicity), the objective function of the proposed model consists in the minimization of the present value of the total system cost:

$$Min\ \ c = \sum_a \frac{1}{(1+\phi)^a} \cdot \sum_n \left\{ \begin{array}{l} \sum_t CI_{t,a} \cdot ng_{t,n,a} \cdot C^U_{t,a} \\ + \sum_t CP_{t,a} \cdot qp_{t,n,a} \\ + CNQ \cdot nq_{n,a} \end{array} \right\} \qquad (1)$$

[2]The Autonomous Communities (CCAA) are the first-level political division of the Kingdom of Spain, established in accordance with the Spanish Constitution.

where:

- The first term is the total investment cost, computed as the product of the unitary investment cost $CI_{t,n,a}$ and the installed capacity, which is the number $ng_{t,n,a}$ of units to be installed multiplied by the capacity $C^U_{t,a}$ of each unit.
- The second term represents the total production cost computed as the product of the unitary production cost $CP_{t,a}$ and the production $qp_{t,n,a}$.
- Finally, the third term represents the cost of the non-supplied demand $nq_{n,a}$.

The optimization problem must be solved subject to the following constraints:

$$qa_{t,n,a} = \begin{cases} Q^I_{t,n} \cdot Ci_{t,n,a} & a = 0 \\ \left(qa_{t,n,a-1} + ng_{t,n,a} \cdot C^U_{t,a} \right) \cdot Ci_{t,n,a} & a \geq 1 \end{cases} \qquad (2)$$

$$0 \leq qp_{t,n,a} \leq qa_{t,n,a} \cdot Di_{t,n,a} \qquad (3)$$

$$\sum_t qp_{t,n,a} + \sum_m f_{m,n,a} + S_{n,a} + nq_{n,a} = D_{n,a} : cm_{n,a} \qquad (4)$$

$$f_{n,m,a} = \frac{\theta_{n,a} - \theta_{m,a}}{X_{n,m,a}} \qquad (5)$$

$$-Q_{n,m,a} \leq f_{n,m,a} \leq Q_{n,m,a} \qquad (6)$$

where:

- (2) calculates the accumulated installed capacity $qa_{t,n,a}$, taking into account $Ci_{t,n,a}$, the annual closure rate of plants that were planned before the first planning year.
- (3) is the maximum production constraint of the generation units, taking into account the availability rate $Di_{t,n,a}$ of each technology. This rate also models the utilization factor for renewable technologies.
  Constraint (3) is essential since it relates investment and operation decisions, which in turns complicates significantly the resolution.
- (4) is the balance between generation and demand considering the international power exchanges and the power flows modeled. The dual variables of these constraints provide the marginal cost $cm_{n,a}$ at each node n.
- (5) calculates the transmission lines power flow $f_{n,m,a}$ along the line (n,m), as a function of the reactance $x_{n,m,a}$ and the phase angles $\theta_{n,a}$ and $\theta_{m,a}$. A reference phase angle $\theta_{ref,a}$ must be chosen such that $\theta_{ref,a}$=0. Each angle must be also upper bounded by 2π rad.
- Finally (6) models the thermal limits of the power flows $f_{n,m,a}$.

B. Stochastic case: uncertain demand

The above minimization problem can be extended to the case of uncertain demand by minimizing the present value of the following expected objective function:

$$Min\ \ c=\sum_a \frac{1}{(1+\phi)^a}\cdot\sum_n\left[\begin{array}{l}\sum_t CI_{t,a}\cdot ng_{t,n,a}\cdot C^U_{t,a}\\ +\sum_s P_{n,s,a}\cdot\left\{\begin{array}{l}\sum_t CP_{t,a}\cdot qp_{t,n,s,a}\\ +CNQ\cdot nq_{n,s,a}\end{array}\right\}\end{array}\right] \quad (7)$$

Indeed, to obtain robust capacities to install, different demand scenarios $D_{n,s,a}$ with different probabilities $P_{n,s,a}$ are considered, although the number $ng_{t,n,a}$ of units to be installed must not depend on index *s*. On the contrary, the rest of variables depend on *s* since they are referred to the system operation, which is different for each demand scenario.

Apart from (2), the following constraints must be taken into account in the stochastic model (instead of (3), (4), (5) and (6)):

$$0\le qp_{t,n,s,a}\le qa_{t,n,a}\cdot Di_{t,n,a} \quad (8)$$

$$\sum_t qp_{t,n,s,a}+\sum_m f_{m,n,s,a}+S_{n,s,a}+nq_{n,s,a}=D_{n,s,a} : cm_{n,s,a} \quad (9)$$

$$f_{n,m,s,a}=\frac{\theta_{n,s,a}-\theta_{m,s,a}}{X_{n,m,a}} \quad (10)$$

$$-Q_{n,m,a}\le f_{n,m,s,a}\le Q_{n,m,a} \quad (11)$$

III. BD Applied to the Capacity Expansion Problem with Network Constraints

Based on (8), a decomposition model that separates the investment and operational decisions is described in this section. The main advantage of this approach is that two separated smaller problems can be solved instead of the original one, which is sometimes too large to be solved with conventional tools.

A. Formulation

This subsection presents the formulation of a bi-level BD for the model described in II.B (see the annex for the mathematical details). To do so, the model has been decomposed as follows:

✓ Investment decisions $ng_{t,n,a}$ and $qa_{t,n,a}$ are optimized in the master problem.

✓ Productions $qp_{t,n,s,a}$, flows $f_{n,m,s,a}$ and phase angles $\theta_{n,s,a}$ are optimized in the sub-problem.

In particular, the master problem objective is:

$$Min\ \ mc=\sum_a \frac{1}{(1+\phi)^a}\sum_{t,n} CI_{t,n,a}\cdot ng_{t,n,a}\cdot C^U_{t,n,a}+rf\left(ng,qa\right) \quad (12)$$

where *rf*(*ng*,*qa*) is the well-known resources function in BD literature, which depends on the master problem decisions $(ng,qa)=(ng_{t,n,a},qa_{t,n,a}, t,n,a)$.

Function *rf*(*ng*,*qa*) is approximated by variable *rf*, an outer approximation using optimality or feasibility cuts (see the annex for more details). If δ^j is a boolean parameter that identifies if a BD cut is an optimality cut (δ^j=1) or a feasibility cut (δ^j =0), at each iteration *j*, then *rf* is obtained using all the cuts until iteration *l*, from the following linear equations, that must be embedded in the master problem:

$$\delta^j\cdot rf\ge RF^j-\sum_{t,n,s,a}\left(\pi^j_{t,n,s,a}\cdot\left(qa_{t,n,a}-QA^j_{t,n,a}\right)\right)\quad j=1,..,l\text{-}1 \quad (13)$$

Apart from these cut constraints, additional restrictions that must be taken into account in the master problem are (2).

The objective of the sub-problem is the resource function *rf*(*ng*,*qa*) expressed as:

$$rf\left(ng,qa\right)=Min\ \ sc=\sum_a \frac{1}{(1+\phi)^a}\sum_{n,s} P_{n,s,a}\cdot\left\{\begin{array}{l}\sum_t CP_{t,n,a}\cdot qp_{t,n,s,a}\\ +CNQ\cdot nq_{n,s,a}\end{array}\right\} \quad (14)$$

At each iteration *l*, the sub-problem constraints are (8), (9), (10) and (11) fixing variables (*ng*,*qa*) at their optimal values from the master problem. Note that since the subproblem constraints do not link variables from different years, each year can be solved independently.

The following subsection describes with more detail this iterative process.

B. Algorithm

The BD's iterative process to solve the proposed expansion model can be described with the following pseudo-code[3]:

1. Parameters initialization:
   - l=1.
   - $L=-\infty$, $U=\infty$.
2. Solve the master problem including (2) and the cuts of (13). Do:
   - $NG^l_{t,n,a}=ng.L_{t,n,a}$
   - $QA^l_{t,n,a}=qa.L_{t,n,a}$
   - $L=rf.L$
3. Solve the sub-problem including (8), (9), (10) and (11), and fixing vector (*ng*,*qa*) as follows:
   - $ng_{t,n,a}=NG^l_{t,n,a}$
   - $qa_{t,n,a}=QA^l_{t,n,a}$
4. New cut building:

[3] "*X.L*" has been used to refer to the optimal value of variable *X*.

- If the sub-problem is unfeasible:
  - Set $\delta^l=0$
  - Solve the following sub-problem to analyze the infactibility of the subproblem:

$$\begin{aligned} & Min \;\; sp = \sum_{t,n,s,a} a_{t,n,s,a} \\ & s.t. \qquad qp_{t,n,s,a} - a_{t,n,s,a} \le qa_{t,n,a} \cdot Di_{t,n,a} \\ & \qquad\qquad (9), (10) \text{ and } (11) \end{aligned} \tag{15}$$

  - $RF^l = sp.L$
  - $\pi^l_{t,n,s,a}$=dual variables of the first constraints of (15).
- else
  - Set $\delta^l=1$
  - $RF^l = sc.L$
  - $U = Min(U, RF^l)$
  - $\pi^l_{t,n,s,a}$=dual variables of (8).

5. Proof of convergence*:* if $1-(L/U) \le \varepsilon$ or $l=IT$ then end. Otherwise, go to step 2.

IV. CASE STUDY

A. System and case description

The case study consists in the application of the proposed model to the Spanish electric system in 2011, considering a time horizon of 9 years (from 2012 to 2020). The lifespan of the plants to be installed has been neglected since it is typically higher than the length of the horizon. It has been assumed that the demand must be completely satisfied, i.e., variables $nq_{n,a}$ and $nq_{n,s,a}$ have been fixed to zero in all the executions.

The Spanish system in 2011 was composed by 82 thermal units which, for this case, have been aggregated in 4 thermal technologies (Nuclear, Coil, Fuel and Combined Cycle). Two additional renewable technologies have been also considered (Wind and Solar; Solar including photovoltaic and thermal). These technologies have been located in 15 nodes of the considered power grid, each node representing a Spanish peninsular CA. The values of the initial install capacity $Q^I_{t,n}$ are shown in Table I for each technology and have been obtained from reports of the Spanish system operator.

TABLE I: INITIAL INSTALLED CAPACITY (MW)

	Nuclear	Coil	Fuel	Combined Cycle	Wind	Solar
Andalucia	0	2051	308	4790	3126	1447
Aragon	0	1342	0	1798	1814	133
Asturias	0	2628	0	0	491	1
Castilla-La mancha	1066	221	948	774	3647	1017
Castilla Leon	466	2707	0	0	5279	393
Cataluña	3142	160	1570	2440	1205	201
C. Valenciana	1085	0	0	2791	1139	274
Extremadura	1957	0	0	0	0	963
Galicia	0	2031	470	1180	3173	12
La Rioja	0	0	0	790	428	69
Murcia	0	0	578	3261	251	345
Navarra	0	0	0	1187	939	130
Pais Vasco	0	217	936	1949	147	22
Cantabria	0	0	0	0	34	2
Madrid	0	0	0	0	0	52

The demand scenarios $D_{n,s,a}$ to be satisfied using the considered technologies (that is, $D_{n,s,a}$ is the total demand minus the productions with hydro, pumping and cogeneration technologies) are based on the historical demand of 2011, increased at an annual rates of 1.6% and 2.6% for 2014 and 2015 respectively and of 6% from 2016 onward. Historical annual rate for 2012 and 2013 due the economical crisis were -2.4 and -2.3% respectively. Table II shows the three demand scenarios considered based on the demand of 2011. They have been labeled as MED, MAX, MIN to identify the cases of medium (MED, demand of 2011), high (MAX, 1.35 times MED) and low demand (MIN, 0.65 times MED) values, respectively.

TABLE II: DEMAND SCENARIOS (MWH/H)

	MAX	MED	MIN
Andalucia	8550	6333	4117
Aragon	2362	1750	1137
Asturias	1785	1322	859
Castilla-La mancha	5602	4150	2697
Castilla y Leon	5523	4091	2659
Cataluña	7634	5655	3676
C. Valenciana	5085	3767	2448
Extremadura	995	737	479
Galicia	4395	3256	2116
La Rioja	625	463	301
Murcia	1469	1088	707
Navarra	1331	986	641
Pais Vasco	2690	1993	1295
Cantabria	524	388	252
Madrid	4536	3360	2184

The probability $P_{n,s,a}$ of each demand scenario has been supposed constant over the time horizon, and independent of each CA, being 25%, 50% and 25% for the MAX, MED and MIN demand scenarios respectively.

The values for the maximum capacity of the transmission network lines have been estimated based on historical data from different public web pages (see Table IV). The network is depicted in Fig. 1.

TABLE IV: MAXIMUM CAPACITY OF TRANSMISSION NETWORK (MW)

	Andalucia	Aragon	Asturias	Castilla-La mancha	Castilla y Leon	Cataluña	C. Valenciana	Extremadura	Galicia	La Rioja	Murcia	Navarra	Pais Vasco	Cantabria	Madrid
Andalucia	-	-	-	550	-	-	-	100	-	-	200	-	-	-	-
Aragon	-	-	-	200	50	550	250	-	-	-	-	200	-	-	-
Asturias	-	-	-	-	550	-	-	-	50	-	-	-	-	250	-
Castilla-La mancha	550	200	-	-	125	-	150	550	-	-	500	-	-	-	5000
Castilla y Leon	-	50	550	125	-	-	-	250	450	250	-	-	3500	250	2000
Cataluña	-	550	-	-	-	-	450	-	-	-	-	-	-	-	-
C. Valenciana	-	250	-	150	-	450	-	-	-	-	600	-	-	-	-
Extremadura	100	-	-	550	250	-	-	-	-	-	-	-	-	-	-
Galicia	-	-	50	-	450	-	-	-	-	-	-	-	-	-	-
La Rioja	-	-	-	-	250	-	-	-	-	-	-	350	250	-	-
Murcia	200	-	-	500	-	-	600	-	-	-	-	-	-	-	-
Navarra	-	200	-	-	-	-	-	-	-	350	-	-	150	-	-
Pais Vasco	-	-	-	-	3500	-	-	-	-	250	-	150	-	200	-
Cantabria	-	-	250	-	250	-	-	-	-	-	-	-	200	-	-
Madrid	-	-	-	5000	2000	-	-	-	-	-	-	-	-	-	-

Fig. 1: Considered Spanish network

Table V presents the reactance of each line (upper diagonal matrix), estimated based on the air distance (lower diagonal matrix) among the capitals of each CA, and using a standard reactance value of 0.01729 pu/100km for 400kV lines (from [26]). Therefore, for simplicity, it has been assumed that each line in the equivalent electrical network has a similar reactance to a single line of 400kV. The reference angle is "Castilla y Leon" in the Northwest of Spain, with a large number of CCAA in its neighborhood.

TABLE V: REACTANCE AND DISTANCES (P.U. AND KM)

	Andalucia	Aragon	Asturias	Castilla-La mancha	Castilla y Leon	Cataluña	C. Valenciana	Extremadura	Galicia	La Rioja	Murcia	Navarra	Pais Vasco	Cantabria	Madrid
Andalucia		-	-	0.06	-	-	-	0.03	-	-	0.06	-	-	-	-
Aragon	-		-	0.06	0.06	0.04	0.04	-	-	-	-	0.02	-	-	-
Asturias	-	-		-	0.04	-	-	-	0.01	-	-	-	-	0.03	-
Castilla-La mancha	324	331	-		0.04	-	0.05	0.04	-	-	0.06	-	-	-	0.01
Castilla y Leon	-	319	212	208		-	-	0.06	0.06	0.04	-	-	0.04	0.04	0.03
Cataluña	-	257	-	-	-		0.05	-	-	-	-	-	-	-	-
C. Valenciana	-	246	-	316	-	304		-	-	-	0.03	-	-	-	-
Extremadura	173	-	-	226	335	-	-		-	-	-	-	-	-	-
Galicia	-	-	50	-	343	-	-	-		-	-	-	-	-	-
La Rioja	-	-	-	-	209	-	-	-	-		-	0.01	0.01	-	-
Murcia	320	-	-	326	-	-	177	-	-	-		-	-	-	-
Navarra	-	138	-	-	-	-	-	-	-	76	-		0.01	-	-
Pais Vasco	-	-	-	-	215	-	-	-	-	48	-	85		0.02	-
Cantabria	-	-	165	-	215	-	-	-	-	-	-	-	114		-
Madrid	-	-	-	68	162	-	-	-	-	-	-	-	-	-	

Investments and operation costs have been estimated based on public sources, assuming an annual increase rate of 2%, equal to the discount rate ϕ (Table VI).

TABLE VI: INVESTMENT (€/MW) AND PRODUCTION COSTS (€/MWH)

	Investment	Production
Nuclear	1500000	9.27
Coil	850000	65.65
Fuel	750000	75.26
Combined Cycle	450000	59
Wind	1200000	0
Solar	1700000	0

Historical international exchanges in 2011 at each node are shown in Table VII (obtained from the Spanish system operator). Expected international exchanges for the years of the planning horizon have been assumed to be equal to those of 2011.

TABLE VII: INTERNATIONAL EXCHANGES OF 2011 (MWH/H)

Andalucia	-513
aragon	-37
Castilla y Leon	-28
Cataluña	105
Extremadura	119
Galicia	-412
Pais Vasco	71

The utilization factors of wind and solar technologies have been fixed to 0.43 and 0.4 p.u. respectively, which can be computed from historical data.

A. Investment planning: stochastic versus deterministic

Fig. 2 shows the investment planning obtained from the stochastic model described in subsection II.B.

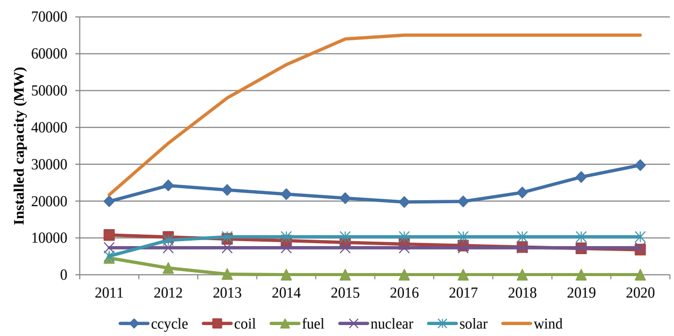

Fig. 2: Stochastic expansion planning by technologies

Note that decommissions for groups installed before 2012 have been modeled (see the resulting fuel evolution), coherent with the current generation technologies trend. Fig. 2 also shows the large increment of the installed capacity of wind technology due to its utilization factor and investment cost, in comparison with solar. To satisfy the demand, combined cycle plants are also installed at the end of the horizon because they recover more costs for the remaining years of the horizon than wind farms, reflecting a very common problem of finite horizon planning. To overcome this drawback, long-term planning analysis with infinite periods is currently being studied by the authors.

Fig. 3 shows the total investment planning obtained from the stochastic model and compared to the corresponding for the three scenarios optimized using the deterministic model described in subsection II.A.

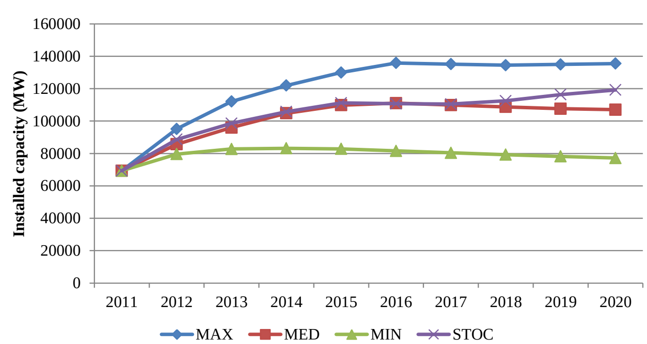

Fig. 3: Comparison between the stochastic and the deterministic total expansion planning

As can be seen from Fig. 3, the expansion planning of the stochastic model is very similar to the deterministic model for MED, anticipating that MED has the higher probability to occur (50%). Planning costs are 178393.5, 105788.1, 45560 M€ for MAX, MED and MIN respectively, and 121901.8 M€ for the stochastic solution. This is quite sensible since stochastic planning is a robust solution against the demand uncertainty, and therefore it has a higher cost than MED (and obviously MIN), but lesser than MAX, since this last consider a high demand scenario with probability 1, which requires too large investments.

An average nodal marginal cost can be computed by weighting the nodal marginal cost of each scenario (MAX, MED and MIN) with the probability of each scenario. Fig. 4 compares those obtained from the deterministic and stochastic resolution approaches by representing their difference. In addition, a black line represents the average of these differences weighted by the demand $D_{n,a}$ of each node. As can be seen the stochastic model leads to a more efficient solution, since these average values trend is positive.

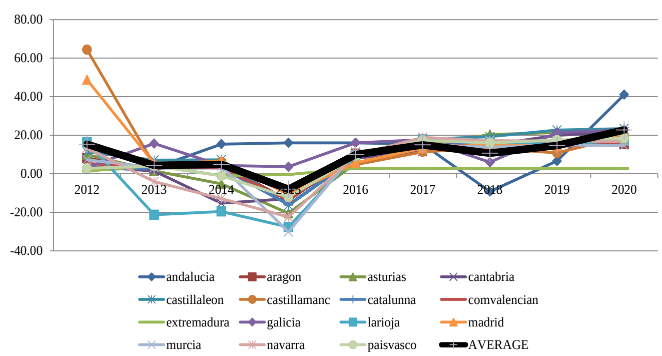

Fig. 4: Marginal costs differences between the stochastic and the deterministic cases

B. Network investments

This subsection shows that network investments should be in accordance with generation investments, to avoid non-supplied demands, but also to take advantages from CCAA climatology. To do so, a large solar penetration is assumed in Andalucía, consisting in installing 300 solar groups each year of the horizon. Under this scenario, Table VI presents the number of installed groups planned by the stochastic model.

TABLE VI: NUMBER OF SOLAR PLANTS IN ANDALUCÍA

		2012	2013	2014	2015	2016	2017	2018	2019	2020
Andalucia	solar	300	300	300	300	300	300	300	300	300
Aragon	wind	300								
Asturias	wind	183								
Cantabria	wind	156								
Castilla y Leon	ccycle	1						1	2	2
	wind	300	300							
Castilla la Mancha	ccycle	4					1	2	3	2
	coil	1								
	wind	300	218							
Cataluña	ccycle							1	2	2
	wind	300	300	1						
C. Valenciana	ccycle					1			1	
	wind	300	300							
Galicia	ccycle								1	1
	wind	300	227	1						
La Rioja	wind	95								
Madrid	ccycle	1								
	wind	300	300							
Murcia	wind	127								
Navarra	wind	151								
Pais Vasco	ccycle									1
	wind	300								

From Table VI, additional groups are installed in the rest of CCAA since imports from Andalucía are bounded by the maximum thermal limits of the lines departing from Andalucía to other CCAA. This means that enforcements in such lines should be planned for a more efficient planning.

Fig. 5 presents the corresponding marginal costs $cm_{n,s,a}$ in Andalucía. As can be seen, prices tend to zero due to the high penetration of the solar technology (even leading to spills).

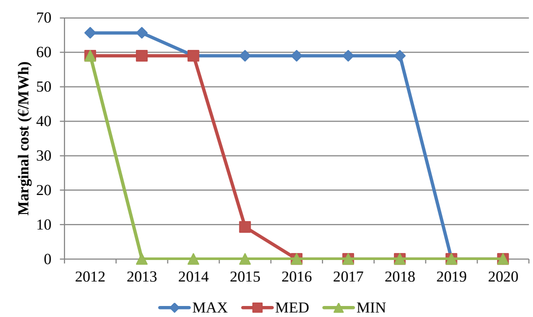

Fig. 5: Andalucía marginal costs vs. new installed solar plants

C. BD algorithm convergence

The executions were run on a 64-bit Inter-Core CPU at 3.4 GHz, programmed in Gams (http://www.gams.com/) and solved using Cplex solver. The problem with all the hours of each year without BD was too large and could not be solved (computer run out of memory). When applying BD a solution was achieved in 99 iterations, with a total execution time of approximately 2 days and 3 hours. Fig. 6 shows the lower and upper bounds of BD algorithm (L and U respectively) at each iteration of the algorithm.

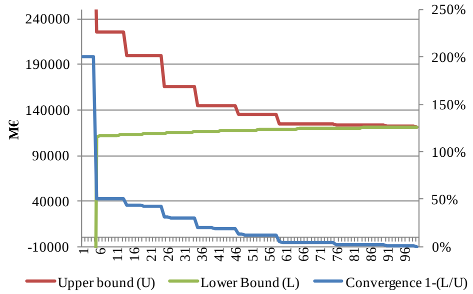

Fig. 6: Evolution of BD algorithm

V. Conclusions

This paper proposes a generation expansion planning model to obtain usable signals for investors in generation assets when the electricity demand is uncertain and DC network constraints without losses are considered. These signals provide the capacity to install and the nodes to locate it for each generation technology.

By considering uncertainty in the demand of the Spanish system, the model has been applied to a simplified network where each node is a Spanish Autonomous Community. Equivalent lines characteristics have been inferred from past historic data. The portfolio includes wind and solar technologies, which have proved to be more competitive than thermal ones, although back-up thermal investments of renewable generation have not been taken into account. Other results of this application prove the robustness and efficiency of the stochastic solution, and also provide insights about the necessary investments in the network when a large amount of renewable generation is installed. It has been also tested that the case study cannot be solved using conventional techniques. BD decomposition is used to solve this drawback.

Future developments may be oriented to apply this methodology for a more recent and longer time horizon, to consider network lines investments for a combined generation and transmission plan, and to improve the power network representation.

Annex: Benders Decomposition

Bi-level BD is used for solving a large-scale problem by means of partitioning it into two separated problems: a master problem (linear, non-linear, and continuous or integer problem) and a sub-problem (linear problem). This partition can be realized by temporal periods, spatial units or scenarios [22].

The problems' structure for which BD can be applied is the following:

$$\begin{aligned} \min_{x,y}\ & c(x)+dy \\ \text{s.t. } & T(x)+Wy=h \\ & x\in X \\ & y\in Y \end{aligned} \tag{16}$$

where $c(x)$ and $T(x)$ are functions of x, d and h are vectors, W is a matrix, and X and Y sets of constraints on x and y respectively, being Y a polyhedron. First block of constraints correspond to the well-known coupled constraints between x and y.

This problem can be represented as a bi-level optimization problem as follows [28]:

$$\begin{aligned} \min_{x,\theta}\ & c(x)+\theta \\ \text{s.t. } & \theta\geq\theta(x) \\ & x\in X \\ & \theta(x)=\min_{y}\ dy \\ & \qquad \text{s.t. } Wy=h-T(x) \\ & \qquad\qquad y\in Y \end{aligned} \tag{17}$$

$\theta(x)$ being the resources function, which is proved to be convex ([28]). The optimization problem in the first level is named master problem, while the second level contains the sub-problem.

Since $\theta(x)$ (second level) is not known a priori, Benders proposes an iterative approximation of $\theta(x)$ by using outer linear functions at each iterative solution $\underline{x}^l$ of the master problem (first level). Each linear function or optimality cut on $\theta(x)$ at $\underline{x}^l$ has the following slope:

$$\left.\frac{\partial\theta(x)}{\partial x}\right|_{x=\underline{x}^l}=\left.\frac{\partial\theta(x)}{\partial(h-Tx)}\right|_{x=\underline{x}^l}\cdot\left.\frac{\partial\left(h-T(x)\right)}{\partial x}\right|_{x=\underline{x}^l}=-\underline{\pi}^l T\left(\underline{x}^l\right) \tag{18}$$

Therefore, the optimality cuts are:

$$\theta\geq\underline{\theta}^l-\underline{\pi}^l T\left(\underline{x}^l\right)\left(x-\underline{x}^l\right) \tag{19}$$

The following scheme describes the iterative algorithm proposed by Benders:

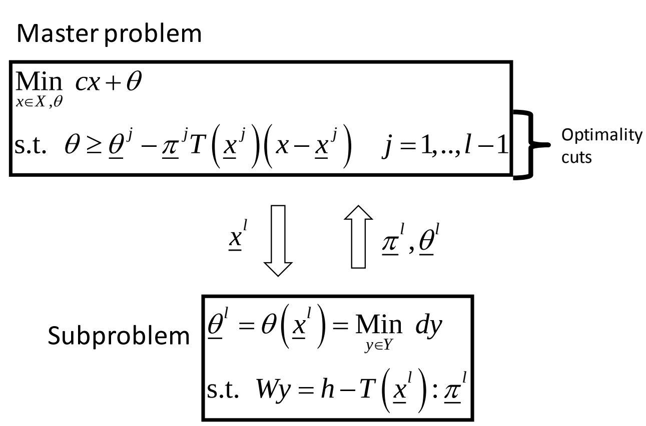

Fig. 7: Benders algorithm

This iterative process converges when the approximation of the recourse function $\theta(x)$ is sufficiently good (that is, when the optimal value of variable θ in the master problem is similar to $\underline{\theta}^j$ for some *j* obtained in the subproblem).

Nevertheless, sometimes the subproblem is unfeasible when fixing a particular solution $\underline{x}^l$ of the master problem (tipically due to the sign of the right-hand side of the subproblem constraints). In this case, it is necessary to broadcast to the master problem feasibility cuts instead of optimality cuts. These new cuts are obtained by minimizing the infeasibilities of the subproblem, i.e.:

$$\begin{aligned} &\nu = \nu\left(x\right) = \underset{y\in Y, a^+, a^- \geq 0}{\mathrm{Min}} \left(ea^+ + ea^-\right) \\ &\text{s.t. } Wy + Ia^+ - Ia^- = h - T\left(x\right) : \varpi \end{aligned} \tag{20}$$

I being the identity matrix and *e* a vector of unitary values. Since $\underline{x}^l$ makes the subproblem unfeasible then $\underline{\nu}^l=\nu(\underline{x}^l)>0$. To avoid $\underline{x}^l$ in the following iteration, a linear approximation of $\nu(x)$ is iteratively built (as with $\theta(x)$) isolating $\underline{x}^l$ by imposing the following linear cut (the feasibility cut):

$$0 \geq \underline{\nu}^l - \underline{\varpi}^l T\left(\underline{x}^l\right)\left(x - \underline{x}^l\right) \tag{21}$$

These cuts are very similar to the optimality cuts (see (19)) but when approximating $\nu(x)$ instead of $\theta(x)$. Since at the end of the algorithm $\nu(x)$ needs to be zero (to avoid infeasibilities), left-hand side of (21) is fixed to a null value, which is the main difference respect to the optimality cuts. The final BD algorithm introducing these feasibility cuts is similar to the one described in Fig. 7 except that now the master problem would include these cuts at each iteration when the subproblem is unfeasible.

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